When the Fermi surface is distorted by warping, the enclosed volume must remain unchanged from the spherical case. For warping of the form (IV-1) Cooper and Raimes [6] give the enclosed volume as

$$V = 4\pi/3\ k_o^3\,[\,1 + .57\,A^2 + 1.85\,A_1^{\ 2}] \quad . \qquad \text{(IV-9)}$$

We shall see later that $|A| < .04$ and $|A_1| < .09$ for all the alkali metals; in addition A and $A_1$ are always negative. Under these conditions the contribution of the terms in $A^2$ and $A_1^{\ 2}$ to the expression in the bracket is less than 2 percent. To a good approximation the volume enclosed by the warped Fermi surface is just that of a sphere with radius $k_o$, and since the enclosed volume must equal that of the Fermi sphere, $k_o$ equals $k_s$ and

$$k_o a/2\pi = .62 \quad . \qquad \text{(IV-9a)}$$

The data of Ham were fitted by choosing a level of energy such that the maximum and minimum values of $ka/2\pi$ for this energy averaged to .62. The values of A, $A_1$, and $k_o$ for this energy were computed using Eq. (IV-1) and values $ka/2\pi$ given by Ham's E vs. k curves. If the value of $k_o a/2\pi$ obtained differed by more than 1 percent from .62 we repeated the procedure for a different value of energy.

In Table 4-1 we tabulate the results of this procedure. Since the actual curves of E vs. k are not shown we give the value of $ak/2\pi$ at the Fermi energy for the principal directions. The last figure on the values for A and $A_1$ is given even though the precision of the fit and of the E vs. k curves used does not justify it; rounding off would obscure some of the changes of warping parameter with lattice constant. In addition, the values of A and $A_1$ for cesium are sensitive to the choice of Fermi energy since k changes very rapidly with energy in this particular case. In Table 4-2 we give values of the warping parameters for lattice constants corresponding to atmospheric pressure and to 15,000 $kg/cm^2$; these values are obtained from a linear interpolation between the values of lattice constant shown in Table IV-1.

Examination of Table 4-1 shows that the k vectors in the 100 and 111 directions are usually equal; for this case (IV-1) leads to the condition $A = .47\,A_1$. We have used this relation in computing B and $B_1$ for values of

$A_1$ between .00 and -.08, using the expressions given in Appendix 1. The result can be expressed as

$$B/A = B_1/A_1 = 3.3 - 60\,A_1, \text{ for } A_1 < 0. \tag{IV-10}$$

C. Dependence of $n^*$ on Pressure

We have obtained a range of values for A and $A_1$ from Ham's work and we can express B and $B_1$ in terms of A and $A_1$. We now notice that the terms arising from the sixth-order Kubic harmonics dominate the expression for $n^*$ Eq. (IV-6). Using the relation (IV-10) for B and setting C = 0 we find that the terms in A and B contribute only about 1 percent to $n^*$ for $|A| \leq .03$. We can simplify the fitting of the data with no significant error by considering only the contribution of terms in $A_1$, $B_1$, and $C_1$ to $n^*$. The expression for $n^*$ then becomes:

$$n^* = 1 + 12.3\,A_1^2 - 24.6\,A_1\,(C_1 - B_1) - .615\,(C_1 - B_1)^2 \tag{IV-11}$$

A non-zero value of $C_1$ can cause $n^*$ to decrease as $|A_1|$ increases. Investigation of the behavior of $n^*$ vs. $A_1$ for different forms of $C_1$ is straightforward but tedious. In Fig. 4-1 we give some curves of $n^*$ vs. $A_1$ obtained using Eq. (IV-11) with the supplementary condition (IV-10) and various forms of $C_1$. The values $C_1 = -.3$ and $C_1 = -.4$ were chosen because they represent the simplest non-zero $C_1$'s and because their magnitude gives values of $n^*$ for $A_1 = 0$ that are in the same range as the observed values in the alkalis. The values $C_1 = -.3 + 4.5\,A_1$ and $C_1 = -.4 + 5\,A_1$ are chosen because the terms in $A_1$ approximately cancel the terms in $A_1^2$ occurring in the expression for $n^*$ and give a steeper initial slope of the $n^*$ vs. $A_1$ curve. Certain features of these curves should be noted.

They show that $n^*$ can decrease as the warping, $|A_1|$, increases; the experimental data and Ham's calculations do not conflict.

The size of the changes in $n^*$ produced by changes in $A_1$ of the magnitude indicated by Table 4-2 is consistent with the size of the observed changes. Looking at the part of the curve before the minimum we see that changes in $A_1$ of .02 produce changes of the order of 10 percent in $n^*$.